Draft version February 1, 2008
Preprint typeset using LATEX style emulateapj v. 04/03/99

PULSAR BRAKING INDEX: A TEST OF EMISSION MODELS?

R.X. Xu[1,2] & G.J. Qiao[2]

Draft version February 1, 2008

ABSTRACT

Pulsar braking torques due to magnetodipole radiation and to unipolar generator are considered, which results in braking index being less than 3 and could be employed to test the emission models. Improved equations to obtain pulsar braking index and magnetic field are presented if we deem that the rotation energy loss rate equals to the sum of the dipole radiation energy loss rate and that of relativistic particles powered by unipolar generator. The magnetic field calculated by conventional way could be good enough but only modified by a factor of ~ 0.6 at most. Both inner and outer gaps may coexist in the magnetosphere of the Vela pulsar.

Subject headings: pulsars: general — radiation mechanisms: nonthermal

1. INTRODUCTION

Pulsar emission process is still poorly understood even more than 30 yr after the discovery. Nevertheless, it is the consensus of the researchers (e.g., Usov 2000) that primary pairs are produced and accelerated in regions (gaps) with strong electric field along magnetic line ($E_\parallel$) and more secondary pairs (with multiplicity $\sim 10^{2\sim 4}$) are created outside the gaps ($E_\parallel = 0$), and that instability may be developed in the secondary $e^\pm$ relativistic plasma in order to give out coherent radio emission. Numerous models have been made concerning gap acceleration, whereas it is urgent to find effective way to test those specific and detailed models against observations.

Because of observational difficulties, only braking indices $n \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2$ (Ω the angular velocity of rotation) of 5 young radio pulsars have been obtained observationally (Lyne & Graham-Smith 1998 and references therein, Camilo et al. 2000). They are: PSR B0531+21 ($n = 2.51 \pm 0.01$), PSR B1509-58 ($n = 2.837 \pm 0.001$), PSR B0540-69 ($n = 2.2 \pm 0.1$), PSR B0833-45 ($n = 1.4 \pm 0.2$), PSR J1119-6127 ($n = 2.91 \pm 0.05$). Certainly, these observed indices include precious information on how pulsars produce radiation. But all of them are remarkably smaller than the value $n = 3$ expected for the pure magnetodipole radiation, according to which, the polar magnetic field strength at pulsar surface, B, is conventionally determined by (e.g., Manchester & Taylor 1977),

$$B = \frac{1}{\sin\alpha}\sqrt{\frac{3Ic^3P\dot{P}}{8\pi^2R^6}}, \qquad (1)$$

where $P = 2\pi/\Omega$ is the rotation period, I the moment of inertia, c the speed of light, R the pulsar radius, α the inclination angle. B is singular (i.e., $B \to \infty$) when $\alpha = 0^{\rm o}$. Therefore, the B-field derivation in this way is questionable and inconsistent since observation indicates $n < 3$, which means other processes do contribute to the braking torque.

Indeed some efforts have appeared to find unusual torque mechanisms to understand the observed braking index (see, e.g., Menou, Perna & Hernquist 2001, and references therein). An alternative effort, within the framework of "*standard*" neutron stars and their magnetospheric emission models, is proposed in this *Letter*. We find that n and B derivation should generally depend on pulsar emission models. Assuming that the orthogonal and aligned parts of magnetic moment are responsible to the dipole-radiation torque and to the unipolar-generator one, respectively, we obtain consistent equations for calculating braking index and magnetic field in the inner vacuum-gap, the space-charge-limited flow, and the outer gap models. We find that all of these models result in braking index $n < 3$, and in return the models could be tested for a particular pulsar if its braking index and the inclination angle are observed.

2. AN ASSUMPTION OF THE TOTAL ENERGY LOSS FOR ROTATION-POWERED PULSARS

Pulsar broad-band emission depends essentially on a complete solution of the formidable well-defined magnetosphere problem in relativistic electrodynamics and plasma physics, which, unfortunately, is still unknown hitherto (e.g., Mestel 2000). Nevertheless, the problem has been understood to some extent in two particular, i.e., the orthogonal and aligned rotating, cases.

Orthogonal Rotator An orthogonal rotator with magnetic dipolar momentum $\mu_\perp$ emits monochromatic electromagnetic waves, the energy loss rate of which is $\dot{E}_{\rm d} = -\frac{2}{3c^3}\mu_\perp^2\Omega^4 \simeq -6.2\times10^{27}\ B_{12}^2\ R_6^6\ \Omega^4$ ergs/s, where $B_{12} = B/(10^{12}{\rm G})$, $R_6 = R/(10^6({\rm cm}))$, and $\mu_\perp = BR^3/2$. These low frequency waves are generally unable to propagate and should be absorbed in neutron star surroundings, and a larger amount of energy and the corresponding momentum could be pumped from neutron stars into their supernova remnants (Pacini 1967).

Aligned Rotator The maximum potential drop in the open-field-line region by unipolar effect is (e.g., Ruderman & Sutherland 1975) $\triangle\Phi = \frac{\mu_\parallel\Omega^2}{c^2} \simeq 5.56\times 10^8\ B_{12}\ R_6^3\ \Omega^2$ cgse. $e^\pm$ pairs (or ions) are accelerated

[1]National Astronomical Observatories, Chinese Academy of Science, Beijing 100012, China
[2]Astronomy Department, Peking University, Beijing 100871, China

in charge depletion gaps, picking up energy in the gaps and angular momentum from the magnetic torque when streaming out. The angular momentum loss requirement (Holloway 1977) can be satisfied if the charged particles can be "attached" to the magnetic field as far as near or out to the light cylinder. Two kinds of gaps are proposed to work in pulsar magnetospheres, which are called as inner and outer gaps. Various inner gaps are suggested, which depend on the binding energy of charged particles in pulsar surface, e.g., the vacuum gap model (Ruderman & Sutherland 1975) with enough binding, the space-charge-limited flow model without any binding (Arons & Scharlemenn 1979, Harding & Muslimov 1998). The outer gap model was suggested to work near the null surface (e.g., Cheng, Ho & Ruderman 1986, Zhang & Cheng 1997) because the charged particles on each side of the surface should flow in opposite direction in order to close a global current in pulsar magnetosphere. It is thus obvious, as seen from above, that the energy loss is model-dependent for aligned rotators, which will be considered when calculating pulsar braking indices and magnetic fields in the next section. Nevertheless, the energy loss rate of an aligned rotator, due to unipolar effect, could be written in the form of $\dot{E}_{\rm u} = -2\pi r_{\rm p}^2 \cdot c\varrho \cdot \Delta\phi$, if a gap has potential drop $\Delta\phi$ and the charge density in the gap is $\varrho = \zeta\varrho_{\rm gj} \approx \zeta\frac{\Omega B}{2\pi c} \simeq 5.3\zeta B_{12}\Omega$ cgse cm^{-3}, where the polar cap radius $r_{\rm p} = R\sqrt{R\Omega/c} \simeq 5.77\times 10^2 R_6^{3/2}\Omega^{1/2}$ cm. $\zeta\sim 1$ since ϱ and $\varrho_{\rm gj}$ are conventionally expected to be in a same order.

The Assumption There are two schools of thought on the energy loss of an oblique magnetized rotator. One group opined that the magnetodipole radiation is the dominate mechanism of braking (e.g., Manchester & Taylor 1977, Dai & Lu 1998, Lyubarsky & Kirk 2001), where no braking appears when $\alpha = 0$. Another group suggested that pulsars' spindown dominates by a longitudinal current outflow due to unipolar generator (e.g., Beskin et al. 1984), where $\Omega =$ constant if $\alpha = 90^{\rm o}$. However, although there are two unseemly points when $\alpha = 0$ for the first school and when $\alpha = 90^{\rm o}$ for the second one, an interesting and strange thing, which is understandable in the next section, is that the derived physical parameters (e.g., B-field strength) are reasonable. We proposed that both energy loss mechanisms above, i.e., via dipole radiation and via unipolar generator, are expected to contribute the total braking torque of an oblique pulsar. Phenomenologically, for a pulsar with a total magnetic momentum $\vec{\mu} = \vec{\mu}_\perp + \vec{\mu}_\parallel$ ($\mu_\perp = \mu\sin\alpha$, $\mu_\parallel = \mu\cos\alpha$), we could write the total energy loss in the form of $\dot{E} = c_\perp\dot{E}_{\rm d} + c_\parallel\dot{E}_{\rm u}$, where $c_\perp$ and $c_\parallel$ are generally two functions of α indicating the contributions of those two energy loss mechanisms, respectively. Certainly $c_\perp(\alpha=\pi/2) = 1$ and $c_\parallel(\alpha = 0) = 1$. An essential and simple assumption to be employed in this paper is $c_\perp = c_\parallel = 1$, since $\dot{E} = \dot{E}_{\rm d} + \dot{E}_{\rm u}$ if $\mu_\perp$ and $\mu_\parallel$ result *independently* in spindowns of $\dot{E}_{\rm d}$ and $\dot{E}_{\rm u}$, respectively. Therefore we have

$$\dot{E} = -\frac{2\mu^2}{3c^3}\Omega^4\eta, \tag{2}$$

with

$$\begin{aligned}\eta &\equiv \sin^2\alpha + 3\cos^2\alpha\frac{\Delta\phi}{\Delta\Phi}\\ &\simeq \sin^2\alpha + 5.4\times 10^{-9} R_6^{-3} B_{12}^{-1}\cos^2\alpha\,\Omega^{-2}\Delta\phi.\end{aligned}$$

3. BRAKING INDEX & ITS IMPLICATION

The energy carried away by the dipole radiation ($\dot{E}_{\rm d}$) and the relativistic particles ($\dot{E}_{\rm u}$) originates from the rotation kinetic energy, the loss rate of which is $\dot{E} = I\Omega\dot{\Omega}$. Energy conservation conduces towards

$$\dot{\Omega} = -\frac{2\mu^2}{3c^3 I}\Omega^3\eta. \tag{3}$$

Based on Eq.(3), the braking index can be derived to be

$$n = 3 + \frac{\Omega\dot{\eta}}{\dot{\Omega}\eta} = 3 + \frac{\Omega}{\eta}\frac{{\rm d}\eta}{{\rm d}\Omega}, \tag{4}$$

which is not exactly 3 as long as η is not a constant. If $\eta\propto\Omega^a$, then $n<3$ for $a<0$ ($n>3$ for $a>0$). For pulsars near death line, $\Delta\phi\simeq\Delta\Phi$, i.e., the maximum potential drop $\Delta\Phi$ available acts on gap. In this case, $\eta = 1+2\cos^2\alpha < 3$, $\dot{\eta} = -2\sin(2\alpha)\dot{\alpha}$. $n<3$ if α gets smaller as pulsar evolves. For pulsars being away from death line, the potential drop $\Delta\phi$ across an accelerator gap, which is model-dependent, is much smaller than $\Delta\Phi$. We discuss baking index in the following models, assuming that $\vec{\mu}$ (μ and α) and I are not changed for simplicity, since both observation (Bhattacharya et al. 1992) and theory (e.g., Xu & Busse 2001) imply that a pulsar's B-field does not decay significantly during the rotation-powered phase.

The vacuum gap (VG) model The basic picture of vacuum gap formed above polar cap with enough binding energy was delineated explicitly in Ruderman & Sutherland (1975), where relativistic primary electrons emit γ-rays via curvature radiation in the gap. The gap potential difference $\Delta\phi_{\rm CR}^{\rm VG} = 4.1\times 10^9\rho_6^{4/7}B_{12}^{-1/7}\Omega^{1/7}$ cgse, where the curvature radius[3] $\rho = \rho_6\times 10^6$ cm. $\rho\simeq\frac{4}{3}\sqrt{Rc/\Omega}\approx 2.3\times 10^8 R_6^{1/2}\Omega^{-1/2}$ for polar cap accelerators. We thus have $\Delta\phi_{\rm CR}^{\rm VG} = 9.2\times 10^{10}R_6^{2/7}B_{12}^{-1/7}\Omega^{-1/7}$ cgse, $\eta_{\rm CR}^{\rm VG}\simeq \sin^2\alpha + 4.96\times 10^2 R_6^{-19/7}B_{12}^{-8/7}\cos^2\alpha\,\Omega^{-15/7}$. For vacuum gap where primary electrons emit γ-rays via resonant inverse Compton scattering off the thermal photons (e.g., Zhang et al. 2000), the potential drop and the η value are $\Delta\phi_{\rm ICS}^{\rm VG} = 1.9\times 10^{13}R_6^{4/7}B_{12}^{-15/7}\Omega^{1/7}$ cgse, $\eta_{\rm ICS}^{\rm VG}\simeq \sin^2\alpha + 1.02\times 10^5 R_6^{17/7}B_{12}^{-22/7}\cos^2\alpha\,\Omega^{-13/7}$.

The space-charge-limited flow (SCLF) model SCLF model works for pulsars with boundary condition of $E_\parallel = 0$ at the pulsar surfaces. The previous SCLF (Arons & Scharlemann 1979) model has been improved to a new version (e.g., Harding & Muslimov 1998) with the inclusion of the frame-dragging effect. Though a simple and general analytical formula for all pulsar is not available in the Harding-Muslimov (1998) model, the potential drop could be well approximated in the extreme cases, regime I and II, which are defined as cases without or with field saturation[4]. In regime II case (i.e., the gap height being larger than $r_{\rm p}$), Zhang et al. (2000) obtained the

[3]Ruderman & Sutherland (1975) supposed there are multipole magnetic fields near pulsar surfaces, and they thus had $\rho_6 = 1$. But in this paper we simply use dipole field lines for indication.

[4]The definitions of regime I and II in Zhang & Harding (2000) have been misprinted (B. Zhang, 2001, personal communication).

potential drop, according to which η values can be calculated. $\Delta\phi^{\rm SCLF}_{\rm II,CR} = 7.1 \times 10^9 R_6^{3/4}\Omega^{1/4}$ cgse, $\eta^{\rm SCLF}_{\rm II,CR} \simeq \sin^2\alpha + 38R_6^{-9/4}B_{12}^{-1}\cos^2\alpha\Omega^{-7/4}$, for the CR-induced SCLF models; $\Delta\phi^{\rm SCLF}_{\rm II,ICS} = 4.2 \times 10^8 R_6^{28/13}B_{12}^{-9/13}\Omega^{18/13}$ cgse, $\eta^{\rm SCLF}_{\rm II,ICS} \simeq \sin^2\alpha + 2.3R_6^{-11/13}B_{12}^{-22/13}\cos^2\alpha\Omega^{-8/13}$, for the resonant ICS-induced SCLF models. In regime I, the stable acceleration scenario should be controlled by curvature radiation (Zhang & Harding 2000), $\Delta\phi^{\rm SCLF}_{\rm I} = 1.8 \times 10^{11}R_6^{4/7}B_{12}^{-1/7}\Omega^{-1/7}$ cgse, $\eta^{\rm SCLF}_{\rm I} \simeq \sin^2\alpha + 9.8R_6^{-17/7}B_{12}^{-8/7}\cos^2\alpha\Omega^{-15/7}$.

The outer gap (OG) model For a self-sustaining outer gap, which is limited by the $e^\pm$ pair produced by collisions between high-energy photons from the gap and soft X-rays resulting from the surface heating by the backflowing primary $e^\pm$ pairs, the potential drop is $\Delta\phi = f^2\Delta\Phi$, where the fractional size of such outer gap $f = 5.5B_{12}^{-4/7}P^{26/21}$ (Zhang & Cheng 1997). $f < 1$, which is satisfied for the five pulsars, if outer gap works. The η value therefore can be calculated, $\Delta\phi^{\rm OG} = 1.59 \times 10^{12}R_6^3B_{12}^{-1/7}\Omega^{-10/21}$ cgse, $\eta^{\rm OG} \simeq \sin^2\alpha + 8.6 \times 10^3B_{12}^{-8/7}\cos^2\alpha\Omega^{-52/21}$.

From these η values in different models, the braking index can be obtained by Eq.(4). For typical pulsars with $R_6 = 1$ and $B_{12} = 1$, we compute the braking index n in each model, which is shown in Fig.1. It is obvious that $n < 3$ as long as inclination angle $\alpha < 90^{\rm o}$ in all of the models. Pulsars with small rotation periods tend to have $n \approx 3$. Also we can see from Fig.1 or Eq.(3) that there is a minimum braking index $n(\alpha = 0^{\rm o})$ for each model. In case of $B_{12} = R_6 = 1$, $n^{\rm VG}_{\rm CR}(\alpha = 0^{\rm o}) = 0.86$, $n^{\rm VG}_{\rm ICS}(\alpha = 0^{\rm o}) = 1.14$, $n^{\rm OG}(\alpha = 0^{\rm o}) = 0.52$, $n^{\rm SCLF}_{\rm II,CR}(\alpha = 0^{\rm o}) = 1.25$, $n^{\rm SCLF}_{\rm II,ICS}(\alpha = 0^{\rm o}) = 2.38$, $n^{\rm SCLF}_{\rm I}(\alpha = 0^{\rm o}) = 0.86$.

We can not solve out magnetic field B by only Eq.(3) because $\eta = \eta(\alpha, \Omega)$. If $\alpha = 90^{\rm o}$ (or $\eta = 1$), the solution of Eq.(3) results in Eq.(1). In principal, Eq.(3) and (4) should be combined to find consistent B and α in case of braking index being known. However, because $1 < \eta < 3$, the magnetic field derived from Eq.(1) is good enough but only modified by a factor $1/\sqrt{\eta} \in (0.58, 1)$.

Based on Eq.(3) and (4), the inclination angles of the five pulsars with observed braking indices are calculated in different models (see Table 1). No solution of α is available for the Vela pulsar (PSR B0833-45) and PSR B0540-69 for the regime II SCLF(ICS) model since their braking indices are smaller than $n^{\rm SCLF}_{\rm II,ICS}(\alpha = 0^{\rm o})$. This is consistent with the fact that these pulsars are young, and their gap heights are thus much smaller than $r_{\rm p}$.

Furthermore, we can determine whether a model works on a particular pulsar by comparing the calculated α in Table 1 with the observed α. Usually α can be derived by fitting the position angle curves of pulsars with high linear polarization in the rotating vector model (Lyne & Manchester 1988). For the five pulsars, only the inclination angle of the Vela pulsar is obtained ($\sim 90^{\rm o}$), however no α value in Table 1 tallies with this observation. There may be two possibilities to explain the discrepancy. (1). The braking torques due to the dipole radiation and to the unipolar generator should be treated and added in an other manner (e.g., Harding et al. 1999), rather than the way of ours. However, our treatment about the torques is reasonable, a further improvement of braking calculation might not change substantially the results presented. (2). No model listed in Table 1 can perfectly describe the actual accelerate situation of the Vela pulsar. The outer gap model explain well the high-energy emission of this pulsar, but could be still a partial description of the global magnetosphere. One possible picture is that both inner and outer gaps coexist in a pulsar's magnetosphere (Usov 2000), but the *interaction* between these two gaps and the pair plasma properties are still very uncertain. It is also possible that pair production process in strong magnetic and electric fields should be improved. For example, if $B > 0.1B_{\rm c}$ ($B_{\rm c} = 4.4 \times 10^{13}$G), γ-photons nearly along curved field lines convert into positroniums which could partially prevent the screening of $E_\parallel$ (resultantly increasing the gap height and possibly having $\zeta > 1$), and therefore the energy loss $\dot{E}_{\rm u}$ increases significantly in polar cap models (Usov & Melrose 1996). Such an increase could result in a larger α in Table 1 (see Eq.(4)) since all magnetic fields of the five pulsar are very strong (near or greater than $0.1B_{\rm c}$). In conclusion, further studies of testing emission models via braking index and of the theoretical meaning of the test result would be interesting and necessary.

4. CONCLUSION & DISCUSSION

We have proposed in this *Letter* that the observed braking index $n < 3$ could be understood if the braking torques due to the dipole radiation and to the unipolar generator are combined. The discrepancy between the observed inclination angle and that derived from the six models of the Vela pulsar in Table 1 may call for improved pulsar emission models. In addition it is found that the magnetic field strength of a pulsar by conventional method could be a pretty good representation of the actual one.

Fig.1 shows the variations of braking index n as functions of pulsar periods. Since pulsars spin down in their life, the curves in Fig.1 represent the variations of n as functions of pulsar ages to some extent. n decreases as a pulsar evolves. However, the Johnston-Galloway's (1999) method to derive braking index can only be applied if n is constant during pulsar life. Therefore n can not been obtained by only P and $\dot{P}$ in principle.

Acknowledgments: We sincerely thank Dr. Bing Zhang for his very helpful discussion, comments and suggestions. The authors wishes to acknowledge the nice computational and academic resources of the Beijing Astrophysical Center. This work is supported by National Nature Sciences Foundation of China (19803001).

REFERENCES

Arons, J., & Scharlemenn, E.T. 1979, ApJ, 231, 854
Beskin, V.S., Gurevich, A.V., & Istomin, Ya.N. 1984, ApSS, 102, 301
Bhattacharya, D., et al. 1992, A&A, 254, 198
Camilo, F., et al. 2000, ApJ, 541, 367
Cheng, K.S, Ho, C., & Ruderman, M. 1986, ApJ, 300, 500
Dai, Z.G., & Lu, T. 1998, A&A, 333, L87
Goldreich, P., Jullian, W. H. 1969, ApJ, 157, 869
Harding, A.K., Contopoulos, I., & Kazanas, D. 1999, ApJ, 525, L125
Harding, A.K., & Muslimov, A.G. 1998, ApJ, 508, 328
Holloway, N.J. 1977, MNRAS, 181, 9
Johnston, S., & Galloway, D. 1999, MNRAS, 306, 50
Lyne, A.G., & Graham-Smith, F. 1998, "Pulsar Astronomy", Cambridge University Press
Lyne, A.G., & Manchester, R.N. 1988, MNRAS, 234, 477
Lyubarsky, Y., & Kirk, J.G. 2001, ApJ, 547, 437
Manchester, R.N., & Taylor, J. 1977, "Pulsars", San Francisco, Freeman
Mestel, L. 2000, ApSS, 272, 283
Menou, K., Perna, R., & Hernquist, L. 2001, ApJ, 554, L63
Pacini, F. 1967, Nature, 216, 567
Ruderman, M. A., & Sutherland, P. G. 1975, ApJ, 196, 51
Usov, V.V. 2000, in IAU Colloq. 177, Pulsar Astronomy–2000 and Beyand, ed. M. Kramer, N. Wex, & R. Wielebinski (ASP Conf. Ser. 202; San Francisco: ASP), 417
Usov, V.V. & Melrose, D.B. 1996, ApJ, 464, 306
Xu, R.X., & Busse, F.H. 2001, A&A, 371, 963
Zhang, B., Harding, A.K., & Muslimov, A.G. 2000, ApJ, 531, L51
Zhang, B., & Harding, A.K. 2000, ApJ, 532, 1150
Zhang, L. & Cheng, K.S. 1997, ApJ, 487, 370

Table 1
The inclination angles (α) of the five pulsars derived from models

Name (PSR)	VG(CR)	VG(ICS)	OG	SCLF(II,CR)	SCLF(II,ICS)	SCLF(I)
B0531+21	2.6^o	2.9^o	5.0^o	2.1^o	1.6^o	33^o
B0540-69	2.5^o	6.3^o	5.2^o	1.8^o	—	36^o
B0833-45	2.6^o	6.9^o	6.7^o	1.0^o	—	31^o
B1509-58	11^o	8.4^o	26^o	7.6^o	2.5^o	81^o
J1119-6127	24^o	6.3^o	52^o	15^o	2.2^o	88^o

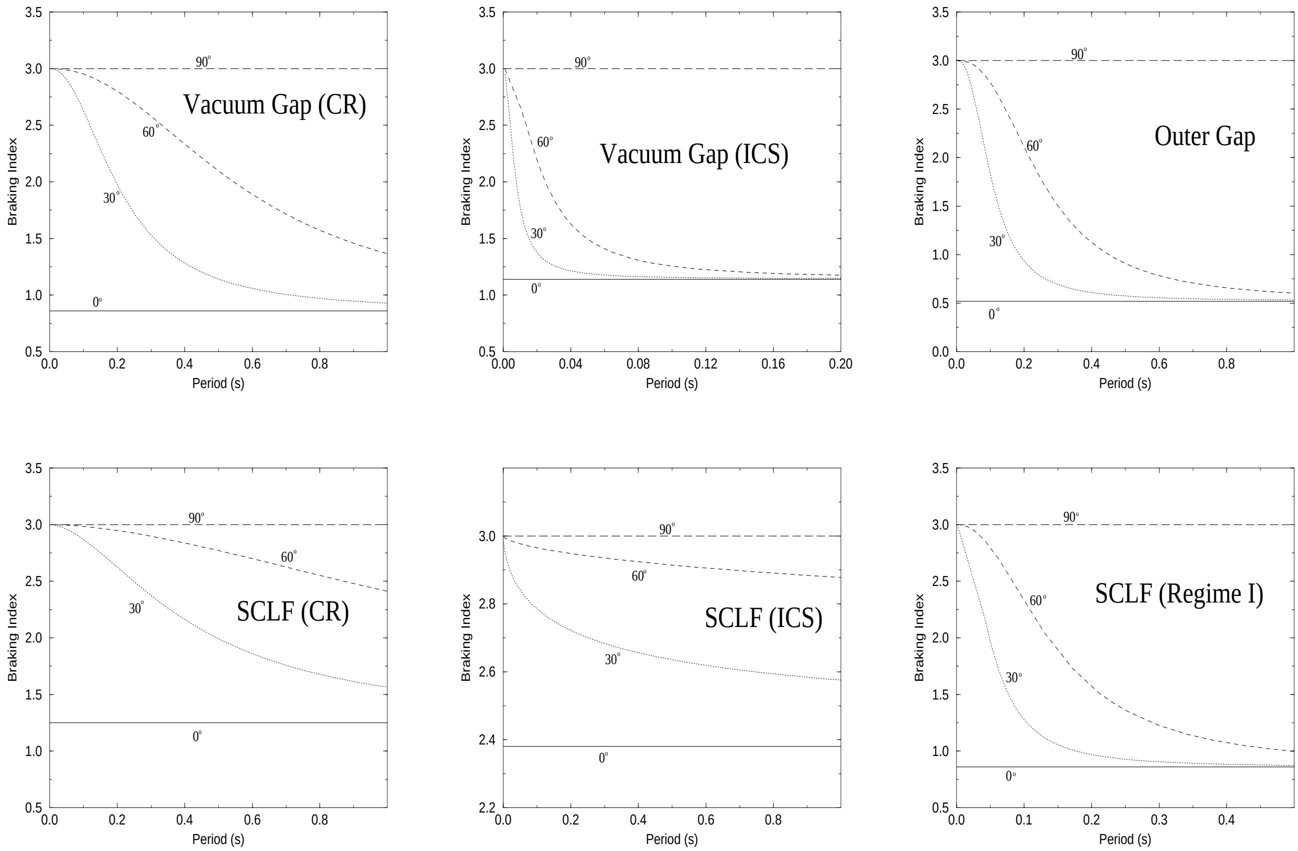

FIG. 1.— A set of calculated braking indices, as functions of rotation period, for six kinds of emission models. Pulsars are assumed to have polar magnetic field $B = 10^{12}$ G and radius $R = 10^6$ cm here. The inclination angles are chosen to be $0^{\rm o}$ (solid lines), $30^{\rm o}$ (dotted lines), $60^{\rm o}$ (dashed lines), and $90^{\rm o}$ (long-dashed lines). "CR" and "ICS" indicate curvature-radiation-induced and resonant inverse-Compton-scattering-induced gaps, respectively. SCLF(Regime I): SCLF model without field saturation, SCLF(II,CR) and SCLF(II,ICS): SCLF model with filed saturation (Regime II).